The electrical conductivity of shock-ionized argon

By H. J. PAIN AND P. R. SMY
Physics Department, Imperial College of Science and Technology, London, S.W. 7

(Received 25 March 1960)

The electrical conductivity of shock-ionized argon has been measured at temperatures between 6000 °K and 13,000 °K and electron number densities in the range 10^{16} to $10^{18}\,\text{cm}^{-3}$. The results are compared with theoretically computed values and show sufficient agreement to be plotted on a linear scale. The experimental technique is based on the measurement of voltages generated in a search coil by the flux from currents induced in a plasma moving rapidly through a magnetic field.

Introduction

The first reported measurements on the electrical conductivity of argon, ionized up to 20 %, moving at velocities $\sim 10^5$ cm/sec in a shock tube and interacting with a magnetic field, were made by Lin, Resler & Kantrowitz (1955). Further measurements have been reported by Patrick & Brogan (1959).

In experiments reported here a method which has some features in common with the technique of Resler *et al.* (1955), but which differs from it in three major respects, is presented. These differences are as follows.

(1) The use of a high-value pulsed magnetic field and the comparison of its effect upon the plasma with that of a low-value d.c. field operating with the same plasma flow conditions.

(2) The choice of a field configuration which provided an axial field uniformly distributed over the area of cross-section of the plasma flow and which allowed the search coil to be positioned at null point, that is, receiving zero total flux from the applied field whilst responding to the field of the currents induced in the plasma. This avoided the use of compensating field coils.

(3) The method of calibration. Details of the shock tube used in these experiments are being reported elsewhere (Pain & Smy 1960). It is a pressure-driven tube with a 5 ft. long O_2–H_2 combustion chamber and a low-pressure channel length of 25 ft. The magnetic field is located at the centre of a Pyrex tube glass section of 2 in. internal diameter and 15 in. length. The length of plasma generated and its thermodynamic properties depend upon the original conditions in the shock tube. Variation of these, notably of the initial shock tube gas pressure, permits a range of shock speeds of Mach numbers between 8 and 23 to be investigated. The gas temperatures and densities behind these shocks may be computed. The length of the luminous plasma, which may be measured from drum camera photographs, varies from more than 50 cm for shocks of low Mach number to 4 cm for the highest speeds.

The magnetic field is formed by passing current through two spirally wound circular coils arranged as in a Helmholtz pair. Its configuration is shown in figure 1. The high-value pulsed field results from the discharge of a 5 kV condenser bank of 900 μF through the coils, the discharge per d of 1 msec being so much greater than the duration of plasma flow through the field that the plasma is presented with a sensibly constant magnetic field and is timed to pass through the field when the field is at its maximum. The field coils are 7 in. in diameter and the peak field value of 32,000 gauss in the plane of each coil is constant over the cross-section of the shock tube to within 10 %. In all other z-planes the variation of B_z over the cross-section of the tube is less than 10 %. This was checked by a subsidiary experiment.

The d.c. field of 100 gauss was generated by connecting the coils across the terminals of a large storage battery in series with a limiting resistance. When the plasma passes through the high-value field, current loops of approximately 2000 amp/cm^2 are induced in the plasma, concentric with the shock tube walls. The flux from these currents generates a voltage in a pick-up coil. This consists of a single turn of 14 s.w.g. copper wire which encircles the shock tube at position z_0 in figure 1, midway between the two field coils.

This voltage is displayed on an oscilloscope trace and the analysis of this trace, together with a calibration experiment, enables the electrical conductivity to be calculated.

Method of measuring the electrical conductivity

The value of the magnetic field at z_0 is zero and the response of the search coil when located at z_0 is less than 10^{-4} of its response when placed in the plane of either field coil. The co-ordinate z defines the direction along the shock tube.

With the search coil at a fixed position z, the large magnetic field is triggered to give a value of $\partial B_z/\partial t$. Variation of the position z allows $\partial B_z/\partial t$ to be plotted for all z from which $B_z(z)$ is calculated. Graphs of B_z and $\partial B_z/\partial z$ against z are shown in figure 1. $\partial B_z/\partial z$ has a maximum value, constant over a distance of 6 cm. The values plotted in figure 1 are those found in the absence of the conducting plasma. It is shown in a later section that the perturbation of these values by the presence of the conducting plasma is negligible, but measurement of the induced field at the position of zero flux of the applied field gives the high sensitivity of a null-point method in a balanced system. Although the value of the magnetic Reynolds number (of the order of 1) implies substantial field distortion the restricted cross-section of the plasma results in considerable wastage of induced flux beyond the plasma boundaries and the field distortion is, in fact, measured to be less than 1 %.

The change of the field with time may be written in the form

$$\frac{DB_z}{Dt} = \frac{\partial B_z}{\partial t} + \frac{\partial B_z}{\partial z}\frac{dz}{dt}.$$

When the field is sensibly constant the current induced in an element of plasma of length dz interacting with the field is given by

$$i \propto \sigma \frac{DB_z}{Dt} dz = \sigma \frac{\partial B_z}{\partial z}\frac{dz}{dt} dz,$$

so that
$$i \propto \sigma u \frac{\partial B_z}{\partial z} dz,$$

where u is the flow velocity and σ is the electrical conductivity of the plasma.

The flux ϕ_E from this current element gives a flux cutting the search coil positioned at z_0 of $k(z)\,\phi_E$, where $k(z)$ depends on the distance between the element and the search coil.

The values of $k(z)$ are found from a calibrating experiment using copper of measured σ_{copper}, in the form of disks of varying thickness having the same cross-section as the inside of the shock tube. With the search coil at z_0 the position of the disk is varied over all z, the plane of the disk being parallel to the plane of the field coils. At each value of z the magnetic field is pulsed and the search-coil response is recorded.

At any z the current generated in the copper is given by

$$i \propto \sigma_{\text{copper}} \frac{DB_z}{Dt} dz = \sigma_{\text{copper}} \frac{\partial B_z}{\partial t} dz,$$

where dz is the thickness of the copper disk,

$$i \propto \phi_E = Q\,\sigma_{\text{copper}} \frac{\partial B_z}{\partial t} dz,$$

where Q is constant. The e.m.f. generated in the search coil is then $k(z)\,[d\phi_E/dt]$. The time integral of this is measured from an oscilloscope trace to give $k(z)\,\phi_E$ for all z, and this, with σ_{copper} and $\partial B_z/\partial t$ known, gives $Qk(z)$ as a function of z. This calibration was carried out with disks of 3, 6, 9 and 12 thousandths of an inch thickness. At the discharge frequency of 1 kc for the current generating the magnetic field the skin depth of copper is approximately 100 thousandths of an inch. The effect of this skin depth increased the rise time for the observed voltage as the thickness of the disks was increased. Each voltage trace was extrapolated to give the value of zero rise time, and, with this correction, the values of $Qk(z)$ for the four thicknesses agreed with each other to within 2 %.

The dimensions of the magnetic field system were scaled so that the values of $Qk(z)$ were significant over the range for which $\partial B_z/\partial z$ was constant at its maximum value but which decreased rapidly with a decrease in $\partial B_z/\partial z$. In effect, the search coil observed flux only from those currents induced in the plasma by the uniform section of the magnetic field gradient. The product $Qk(z)\,[\partial B_z/\partial z]$ is now plotted against z (figure 1). The contribution to the flux through the search coil from a plasma element of thickness dz at z is given by

$$Qk(z) \frac{\partial B_z}{\partial z} \frac{dz}{dt} \sigma\, dz,$$

and for all z beyond the constant $\partial B_z/\partial z$ region this contribution is small.

The total contribution to the flux through the search coil by a succession of elements in the form of a plasma slug moving through the interacting field in effect integrates

$$Qk(z) \frac{\partial B_z}{\partial z} \frac{dz}{dt} \sigma\, dz$$

with respect to z over the length of the plasma slug. If V is the voltage induced in the search coil,

$$\int V\,dt = \int Qk(z)\frac{\partial B_z}{\partial z}u\,dz.$$

For a plasma slug which is long compared with the field region the limits of integration on the right-hand side are from z to $-\infty$, when the front of the plasma is at z. Now

$$\frac{d}{dt}\int V\,dt = V = \frac{dz}{dt}\frac{d}{dz}\int Qk(z)\frac{\partial B_z}{\partial z}a\,dz$$

$$= u^2 Qk(z)\frac{\partial B_z}{\partial z}\sigma.$$

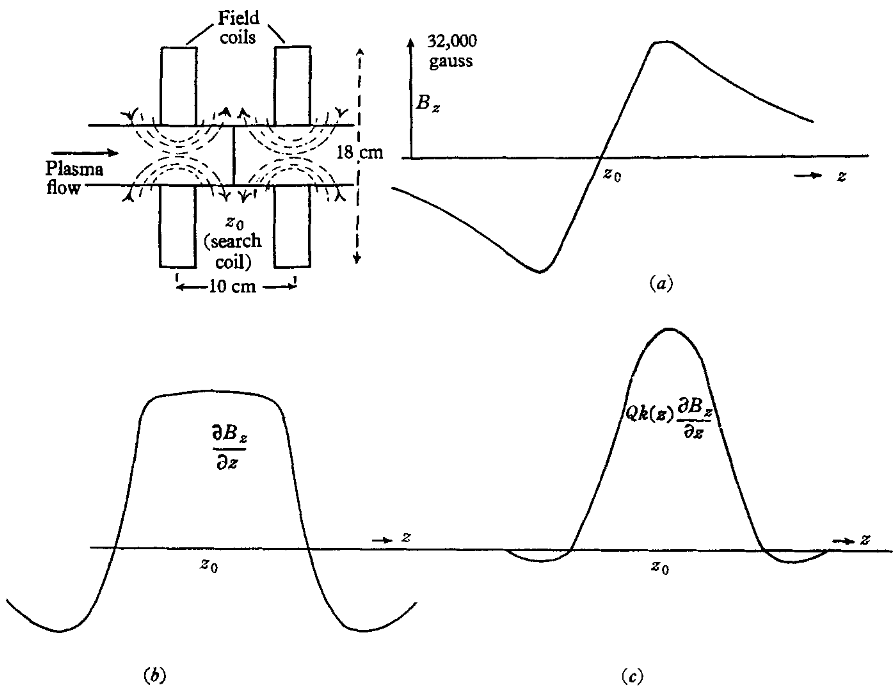

FIGURE 1. Field coil configuration showing search coil position, field lines and direction of plasma flow in the shock tube. (*a*) Graph of B_z versus z. (*b*) Graph of $\partial B_z/\partial z$ versus z. (*c*) Graph of $Qk(z)\,[\partial B_z/\partial z]$ versus z. $Qk(z)$ relates the current induced in the plasma with its magnetic flux cutting the search coil.

The value of u^2 is written here as a close approximation to the correct expression uu', where u' is the shock-front velocity dz/dt. The error introduced by this approximation is about 10 %.

The voltage pick-up is therefore the curve $Qk(z)\,[\partial B_z/\partial z]$ multiplied by some constant value of $u^2\sigma$.

The total trace consists of a curve of this type associated with the leading edge of the slug separated by a distance depending on the length of the slug from an inverted curve of the same shape representing the trailing edge. This was verified experimentally.

The total field generated in the search coil had a maximum value of 300 gauss when the high-value interacting field was used and its effect on this field was negligible (only 1 %).

Calculation of the conductivity

A plasma slug of length L is considered to have reached the position where it is centrally located about z_0. L is measured from the interval between the two peaks of the oscilloscope trace illustrated in figure 2*b*. Then

$$\int V dt = u\sigma \int_{\frac{1}{2}L}^{-\frac{1}{2}L} Qk(z) \frac{\partial B_z}{\partial z} dz,$$

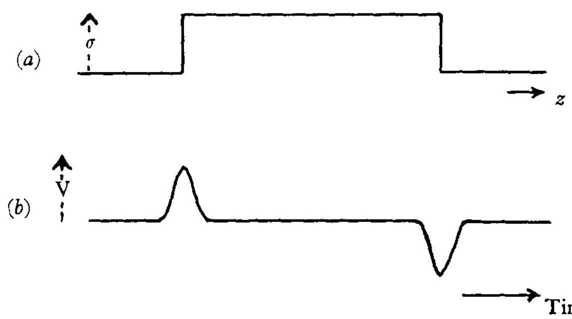

FIGURE 2. (*a*) The electrical conductivity of shock-ionized argon represented as a step function extending from the shock front to the contact surface. (*b*) The search-coil response expected when the plasma flows through the magnetic field.

where the upper time limit in the integration is that time at which the plasma has reached the point where its length is centred about z_0 and this corresponds to the point on the trace of figure 2*b* midway between the positive and negative pulses. The integral

$$\int_{\frac{1}{2}L}^{-\frac{1}{2}L} Qk(z) \frac{\partial B_z}{\partial z} dz$$

is plotted for all L and from this graph and the area $\int V dt$ the value of $u\sigma$ is found. The area $\int V dt$ is found for the positive pulse and for the negative pulse associated with the exit of the plasma from the field region. The mean of these two values averages the value of the electrical conductivity over the length of the plasma slug.

Discussion of search-coil response

If the conducting gas is considered as a uniform slug extending from the shock front to the contact surface then the conductivity function (*a*) and the search-coil response (*b*) are as shown in figure 2.

Increasing the shock Mach number with a constant channel pressure reduces the distance between the shock front and the contact surface. Reducing the channel pressure with constant Mach number also reduces this distance. The variation of the length of conducting gas for low shock Mach number with high channel pressure and high Mach number with low channel pressure is shown in the drum camera records of figure 3. As the shock Mach number increases with reduced channel pressure the peaks of the search-coil response trace move together.

At channel pressures greater than or equal to 3 mm Hg the separation of the peaks of the trace are in good agreement with the length of the luminous gas

$P_1 = 0{\cdot}1$ mm $\qquad$ $P_1 = 0{\cdot}2$ mm $\qquad$ $P_1 = 1$ mm

FIGURE 3. Drum camera records showing variation of the length of luminous gas with initial shock tube pressure (P_1). The synchronizing spark triggers the oscilloscope which records the search-coil response. The search coil is seen as a narrow vertical line midway between the two field coils.

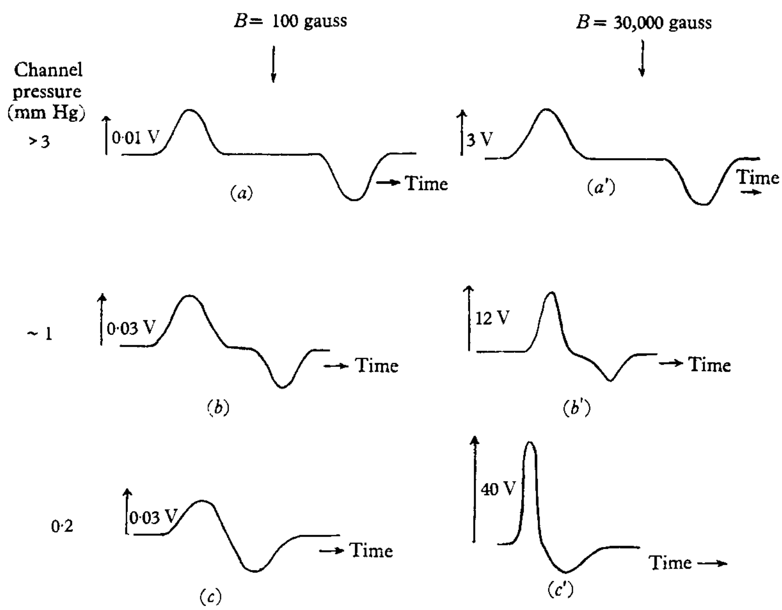

FIGURE 4. Search-coil response traces obtained when shock-ionized argon flows through a magnetic field. The traces (a) for $B = 100$ gauss and (a') for $B = 30{,}000$ gauss show that, for high electron number densities, the conductivity may be represented by figure 2a. As the number density of the plasma is reduced the conductivity profile is distorted until, at very low densities and high field value (trace c'), the symmetry of the trace is completely destroyed. Scale of volts per gauss of induced field is constant for all traces so that differences in pulse height reflect variation of conductivity.

measured from the drum camera records, and the curves resulting from the leading and trailing edges of the conductivity pulses are symmetric. Such a trace for a weak interacting field of 100 gauss is shown in figure 4a. This suggests that the conductivity is closely represented by the step function in figure 2a. There is a small but finite separation between the shock front and the luminous gas front associated with the ionization relaxation time. The shock front is often seen in drum camera photographs as a narrow bright line formed by impurities ionized at the shock front ahead of the luminous front.

Figure 4*b* is the trace obtained for a field of 100 gauss and a channel pressure of 1 mm Hg. Some departure from symmetry is evident here, the peak of the leading pulse being greater than that of the trailing edge. This may be associated with a steep rise of the ionization and electrical conductivity to that value appropriate to the luminous shock front. Behind this front both ionization and electrical conductivity decay to a lower value at the trailing edge of the luminous gas. Further reduction of channel pressure to 0·2 mm Hg restores the symmetry as shown in figure 4*c*. This does not indicate a restored step function for the conductivity profile since this response is to be expected from a slug of ionized gas shorter than the extent of the interacting field region.

This may be seen from the following argument. The flux contribution from such a short slug of plasma will be proportional to that element of area under the curve of $Qk(z)$ $[\partial B_z/\partial z]$ standing on a base of constant width equal to the length of the slug. As the slug moves along the z-axis of the curve $Qk(z)$ $[\partial B_z/\partial z]$ the magnitude of the flux contribution follows a curve of similar shape when plotted against the distance z or the time t. It is the differential of this curve that determines the shape of the response trace and this differential has the form of the curve shown in figure 4*c*.

At high values of the interaction field, i.e. 30,000 gauss, the response is similar to that at low fields when the channel pressure is greater than or equal to 3 mm Hg (figure 4*a*′). Reducing this pressure to 1 mm Hg, however, produces an asymmetry far more pronounced than in the case of the low field as is shown by figure 4*b*′. The final trace, figure 4*c*′, for a field of 30,000 gauss and initial channel pressure of 0·2 mm Hg shows the extreme limit of this distortion. The effect of strong field interaction on flows of reduced density may therefore be seen as a reshaping of the conductivity profile. This is discussed in more detail in the following section.

Discussion on experimental results

Most of the experiments made by Lin *et al.* (1955) were carried out at 1 cm Hg initial channel pressure. A large number of their experiments failed to achieve the equilibrium value of the electrical conductivity which resulted in the search coil giving a maximum response at the trailing edge of the trace. This meant that the value of the electrical conductivity was still rising at the rear of the luminous plasma. In the experiments reported here the leading peak of the trace was always equal to or greater than the peak associated with the trailing edge. This would be expected when the electrical conductivity at the front of the plasma was equal to or greater than that at the rear. Where the equilibrium condition was reached in the experiments of Lin *et al.*, the results agreed with the theory of Spitzer & Harm (1953) for a highly ionized gas. This theory is sufficient for $\alpha > 10^{-3}$ (α is the degree of ionization), that is, $T > 8000$ °K, where the dominant mechanism for energy exchange is the distant encounters between electrons and ions.

At channel pressures of 10 cm Hg the conductivity results of Lin *et al.* fell below the predicted values of Spitzer & Harm, but when the electrical conductivity reached the equilibrium value, they matched a modified theory of Lin *et al.* in which a second term was introduced to allow for close encounters between electrons and neutral atoms in a slightly ionized gas. The trend of the non-equilibrium conductivity results at 10 cm channel pressure followed this modified theory, but individual values were consistently below it.

The annular experiments of Patrick & Brogan (1959) were made at 1 mm Hg constant channel pressure where, for a high field, the scalar relation for the current density, $j = \sigma u B$, might be expected to break down. When $\omega_e \tau_e \sim 1$, where ω_e is the angular frequency of electron gyration around the magnetic field lines and τ_e is the period between collisions, the theory of Schlüter (1950) shows that the conductivity may be expected to be less than that given by $j = \sigma u B$. The major contribution to this effect, when the magnetic field has a large component along the flow direction, is due to the electron making a complete gyro orbit between collisions. The resulting Hall potential in the $\mathbf{j} \times \mathbf{B}$ direction gives a current if a path is provided. If this Hall current is allowed to flow the resulting conductivity is given by

$$\sigma = \frac{\sigma_0}{1+(\omega_e \tau_e)^2},$$

where σ_0 is the electrical conductivity in the absence of a magnetic field. The results of Patrick & Brogan support this relation for various values of $\omega_e \tau_e$.

The results of the present experiments are plotted on two continuous curves each of which corresponds to a constant pressure of O_2–H_2 mixture in the combustion chamber before ignition. In figure 5 this constant pressure is $2\frac{1}{2}$ atm and in figure 6 it is $7\frac{1}{2}$ atm. With this pressure fixed, the shock Mach number and temperature variation are obtained by changing the initial pressure in the shock tube and these variables are shown as abcissae plotted against the conductivity as ordinate. The Mach number measurements showed a variation of 3 % when the initial O_2–H_2 pressure was $2\frac{1}{2}$ atm and less than 3 % when the O_2–H_2 mixture was $7\frac{1}{2}$ atm. The dotted curves show the values of the conductivity computed by DeLeeuw (1958) using the theory of Spitzer & Harm (1953) modified by Lin *et al.* (1955) for close encounters.

The pairs of points associated with a given Mach number represent one value of the conductivity obtained when using the pulsed field of 30,000 gauss and another using the same flow conditions and a d.c. field of 100 gauss, but neither field gives consistently the higher or lower value of any pair and no distinction is drawn between them in figures 5 and 6.

When the initial channel pressure is above 1 mm Hg the experimental and theoretical values show more satisfactory agreement in figure 6 than in figure 5. Below this pressure, however, experimental values on both curves become progressively lower than those given by the Spitzer & Harm theory. This cannot be satisfactorily explained as a simple consequence of the non-scalar conductivity effect because the same value of conductivity at $M = 22$ is obtained for $B_z = 100$ gauss as for $B_z = 30{,}000$ gauss. The large field has a radial component $B_r = 6000$ gauss which gives $\omega_e \tau_e \sim 1$ so that a Hall potential is developed in the

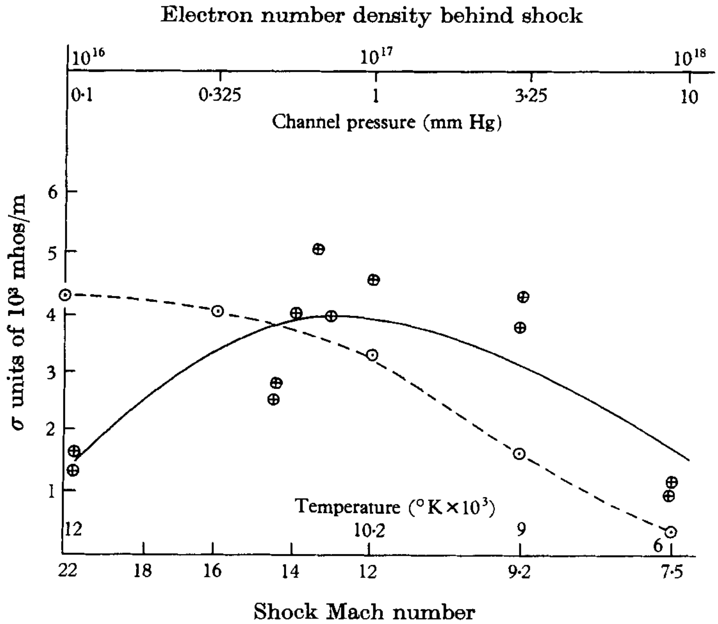

FIGURE 5. The electrical conductivity of shock-ionized argon versus temperature at electron number densities between 10^{16} and $10^{18}/cm^3$. Pre-ignition pressure of O_2–H_2 mixture in the combustion chamber is $2\frac{1}{2}$ atm. Variation of the initial shock tube pressure controls the change of shock Mach number and temperature, all other conditions being unchanged. – – ⊙ – –, Theoretical curve (Spitzer & Harm) modified by Lin *et al.*; ⊕, experimental values.

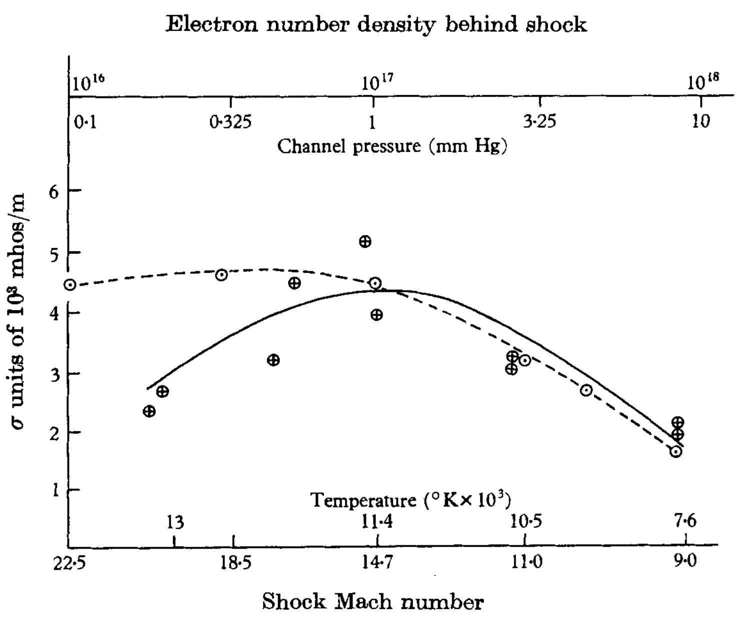

FIGURE 6. As for figure 5, but with pre-ignition pressure of O_2–H_2 mixture equal to $7\frac{1}{2}$ atm. – – ⊙ – –, Theoretical curve (Spitzer & Harm modified by Lin *et al.*), ⊕, experimental values.

$(\mathbf{j} \times \mathbf{B}_r)$ direction. This is not the case for $B_z = 100$ gauss. The search-coil responses for these points, figures 4c and 4c', show quite distinctly that the electrical conductivity reached its equilibrium value in each case.

An explanation of this result may be postulated as follows. The method of obtaining the conductivity by integrating the area under the search-coil response trace gives a mean value of σ over the plasma length. Where the profiles of σ and search-coil response are given by figure 2 a and b this method is valid. When the search-coil response trace is distorted from 2b a more detailed analysis of the trace is required.

Distortion of the profile at low gas densities and higher interaction field values may be considered in terms of the following mechanisms which contribute to changes of conductivity: (1) localized Joule heating from currents induced in the plasma; (2) distortion of the fluid flow pattern as a result of the interaction and plasma heating from magnetic body force compression; (3) non-scalar conductivity, $\omega_e \tau_e \sim 1$.

All heating effects increase the conductivity and this accounts for the high value of the search-coil response (note that all traces in figure 4 have the same volts per gauss scale).

The distortion of the fluid flow pattern in these interactions has been studied in considerable detail and it has been found that the strength of interaction may be described in terms of a parameter involving the ratio of the square of the Lundquist number $R_1 = \sigma B_z z \sqrt{(4\pi\mu/\rho)}$, where z = length of interaction region, μ = permeability, ρ = gas density, to the magnetic Reynolds number

$$R_m = 4\pi z \mu u \sigma.$$

A wide range of values of this parameter has been investigated and the results are being published Pain & Smy (1960).

When $B_z = 100$ gauss no question of flow distortion arises. When $B_z = 30{,}000$ gauss the value of the significant component B_{radial} is about 6000 gauss. This field value, together with an initial channel pressure of 3 mm has been found to be the limit at which distortion of flow is not a serious effect. At pressures below 3 mm, flow distortion is likely.

The energy contribution to the plasma from Joule heating is

$$\int \frac{j^2}{\sigma} dt = \int \sigma u B_r^2 \, dz$$

and this is lost from the kinetic energy of the plasma. In addition to the magnetic force opposing the flow there is a radial compression by a force $\sigma u B_r B_z$.

If Hall currents flow in the z direction (as in Patrick & Brogan 1959), the third mechanism reduces the conductivity by a factor of 2 or more so that the final value of the conductivity is seen to be the result of a number of conflicting influences.

At the moment, although all the above factors may contribute to a distortion of the search-coil response, a rigorous treatment of their total effect upon the plasma is a matter of some difficulty which has not been attempted. The values of the electrical conductivity are therefore open to question in conditions where the oscilloscope trace departs from symmetry. Where this difficulty does not arise the theoretical and experimental values show quite reasonable agreement considering the difficulties of obtaining perfect reproducibility in the operation of a shock tube when attempting absolute measurements.

The authors are grateful to Professor P. M. S. Blackett for his continued interest and encouragement in this work.

Discussions with Dr Latham and other members of the High Temperature Group have been of great help. Finally, it is a pleasure to acknowledge the work of Mr A. H. Chapman who is responsible for the operation and maintenance of the shock tube.

One of us, P. R. Smy, has carried out this work whilst in receipt of a maintenance grant from D.S.I.R.

REFERENCES

DeLeeuw, J. H. 1958 *Institute of Aerophysics, University of Toronto, Rep.* no. 49.
Lin, S. C., Resler, E. L. & Kantrowitz, A. 1955 *J. of Appl. Phys.* **26**, 95.
Pain, H. J. & Smy, P. R. 1960 *Proc. Phys. Soc.* (in the Press).
Patrick, R. M. & Brogan, T. R. 1959 *J. Fluid Mech.* **5**, 289.
Schlüter, A. 1950 *Z. Naturforsch.* **5***a*, 72.
Spitzer, L. & Harm, R. 1953 *Phys. Rev.* **89**, 977.